

COUPLINGS IN THERMOPOROMECHANICS : HOW TO DISTINGUISH THE IMPORTANT FROM THE UNNECESSARY?

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INTRODUCTION

Couplings [1] between rock, saturating fluid and temperature are an essential step to modelise correctly the mechanical behaviour of deep rocks. Nevertheless, in certain practical applications, the general thermoporoelastic problem can be strongly simplified, the state variables being decoupled.

STATE LAWS OF THERMOPOROELASTICITY

The relevant description of a thermoporoelastic transformation is based on the existence of three state variables ϵ (strain tensor), T (temperature) and m (mass of injected fluid by unit of total initial volume) [2,3].

The first state law (thermoporous Hooke's law) relates the stress components σ_{ij} to the three state variables

$$\sigma_{ij} - \sigma_{ij}^0 = \left[K_B - \frac{2G}{3} \right] \epsilon_{kk} + 2G\epsilon_{ij} - \alpha(p - p_0) - \alpha_B K_B (T - T_0) \quad (1)$$

in which G , K_B , α and α_B are respectively the shear modulus, the drained bulk modulus, the Biot's coefficient and the drained thermal expansion coefficient. The second state law relates the pore pressure of the fluid to the three state variables

$$p - p_0 = -\frac{K_u - K_B}{\alpha} \epsilon_{kk} + \frac{K_u - K_B}{\alpha^2} \left(\frac{m}{\rho_0} \right) + \frac{\alpha_u K_u - \alpha_B K_B}{\alpha} (T - T_0) \quad (2)$$

K_u and α_u are the undrained bulk modulus and thermal expansion coefficient and ρ_0 the fluid density in the reference state. The third state law relates the total entropy of the system to the three state variables that is

$$s = s_0 + \alpha_u K_u \epsilon_{kk} + C_\epsilon m_0 \frac{T - T_0}{T_0} + m s_m^0 - \left(\frac{m}{\rho_0} \right) \frac{\alpha_u K_u - \alpha_B K_B}{\alpha} \quad (3)$$

C_ϵ being the specific heat under isochoric (no variation of volume) conditions.

CONSERVATION, DIFFUSION AND DIFFUSIVITY LAWS

To take into account the evolution of the state variables over time, two diffusion laws has to be introduced : Darcy's law for fluid and Fourier's law for temperature. By combining these diffusion laws with conservation laws (mass and energy balances), one obtains the generalised diffusivity equations that is

$$\frac{1}{\eta} \frac{\partial p}{\partial t} + \alpha \frac{\partial \epsilon_{kk}}{\partial t} - \frac{L\rho_0}{T_0\eta} \frac{\partial T}{\partial t} = \frac{k}{\mu} \nabla^2 p \quad (4)$$

for the pore pressure (k is the intrinsic permeability and μ the dynamic viscosity of the fluid) and

$$(\alpha_u K_u T_0 - L\rho_0 \alpha) \frac{\partial \epsilon_{kk}}{\partial t} - \frac{\rho_0 L}{\eta} \frac{\partial p}{\partial t} + \left(\frac{L^2 \rho_0^2}{T_0 \eta} + C_\epsilon m_0 \right) \frac{\partial T}{\partial t} + T_0 \vec{M} \vec{V} s_m = \kappa \nabla^2 T \quad (5)$$

for the temperature (κ is the thermal conductivity). L and η are combinations of thermoporoelastic constants and s_m , the specific entropy of the fluid.

In the general case, a boundary thermoporoelastic problem contains 17 unknowns (6 stress components, 6 strain components, 3 displacements, one pore pressure and one temperature). 17 equations are required : 3 equilibrium equations, 6 thermoporous equations, 6 compatibility equations, and two diffusivity equations. Let us show now that the problem can often be decoupled.

EXAMPLE : WATERFLOODING AROUND A WELLBORE

We consider a slice of poroelastic axisymmetric reservoir (inner radius R , outer radius R_d and height h). The well is filled by a fluid (identical to that of the reservoir) at temperature T_f while the reservoir is at temperature T_R . At the wellbore, one imposes a constant flow rate Q .

General solution (finite element)

The set of equation is implemented in a finite element code [4] to simulate the fully coupled thermoporoelastic problem. In the example presented in *figs 1 and 2*, the injected fluid is supposed to be 100°C below the reservoir temperature. The flow is purely radial and the upper and lower boundaries are impermeable. The initial stress is zero everywhere in the domain. The problem is solved under plane state of strain ($\epsilon_{zz}=0$). The main results can be summarised as follows:

- convection has a strong effect on the temperature profile which propagates much faster when the convection term is taken into account (*fig. 1*), but the pore pressure and the volumetric strain have only negligible effects on temperature,

- temperature has no effect on pore pressure which appears independent of the thermal transfer and is only governed by Darcy's law. Given the high value of the permeability, the pressure profile reaches very quickly a stationnary state (*fig. 2*).

At the waterflooding time scale, the coupling between pressure and temperature only exists through the convective term in the sense pore pressure temperature. In brief, one can consider that the specific entropy of the fluid s_m and the total entropy of the system s are only temperature dependent.

Decoupling strain/pore pressure

Let us consider now a purely isothermal problem. Given the radial symmetry and the plane strain hypothesis, the general system is reduced to 7 unknowns $\sigma_{\rho\rho}$, $\sigma_{\theta\theta}$, σ_{zz} , $\epsilon_{\rho\rho}$, $\epsilon_{\theta\theta}$, and p . In that case, one can easily show that the pore pressure is decoupled from the strain. Indeed, substituting the Hooke's equations (1) in the equilibrium equation taking account of the compatibility equation, the hydraulic diffusivity equation becomes

$$\frac{\partial p}{\partial t} = \frac{k}{\phi\mu C_t} \nabla^2 p \quad \text{with} \quad C_t = \frac{\alpha^2}{K_B + \frac{4G}{3}} \quad (6)$$

Evolution towards a simple analytical model

Assuming a stationary state for the pore pressure and neglecting the conductive term with respect to the convective the temperature equation becomes (in polar coordinates)

$$m_0 C_\epsilon \frac{\partial T}{\partial t} = - \frac{\rho_0 C_f Q}{2\pi h \rho} \frac{\partial T}{\partial \rho} \quad (7)$$

(7) can be easily solved by Laplace transformation [5] and leads to the Heavyside solution

$$T(\rho) - T_R = (T_f - T_R) H(\rho_{inf} - \rho)$$

with $H(x) = 0$ if $x < 0$ with $\rho_{inf} = \sqrt{\frac{C_f}{C_\epsilon} \frac{1}{\pi h} t}$

$$H(x) = 1 \quad \text{if} \quad x \geq 0 \quad (8)$$

The knowledge of the temperature evolution and the complete decoupling of the problem allow us to calculate the thermal stress component by using the method of Goodier's potential [6]. In particular, the tangential component $\sigma_{\theta\theta}$ is such that

$$\sigma_{\theta\theta} = - \frac{E_B}{1 + \nu_B} \frac{1}{\rho^2} \int_R^\rho W(\rho, t) \rho d\rho + \frac{E_B}{1 + \nu_B} W(\rho, t)$$

with $W(\rho, t) = \frac{1 + \nu_B}{1 - \nu_B} \alpha_B [T(\rho, t) - T_R]$

$$(9)$$

The thermal induced stress "follows" the evolution of the temperature (fig. 3) : the stress component is relaxed in the cooled region but (because of static equilibrium!) compressed in the hot region. Waterflood has therefore a tendency to reduce the fracturing pressure. This phenomenon is known as "thermal induced fracturing".

References

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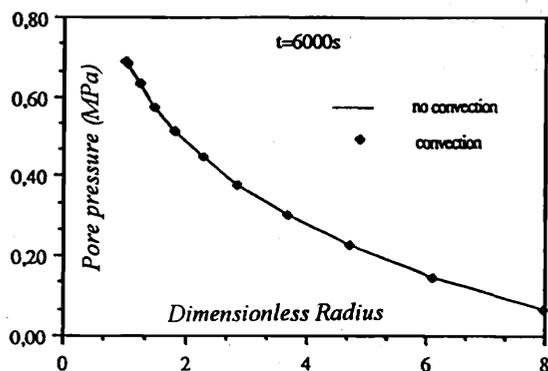


fig. 1 Effect of temperature on pore pressure

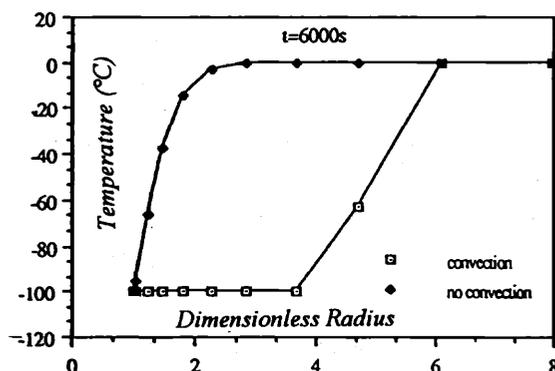


fig. 2 Influence of convection on temperature field

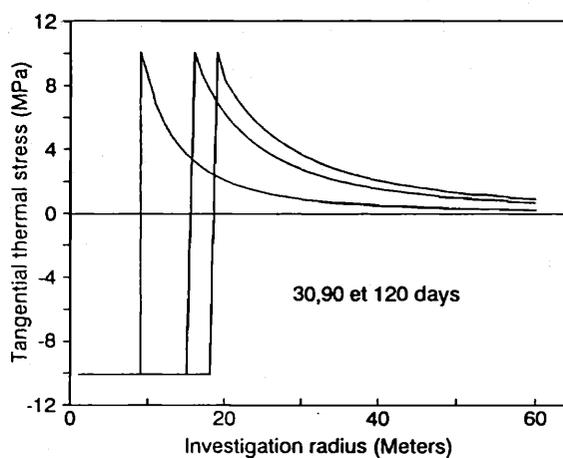


fig. 3 Evolution of thermal tangential stresses during waterflooding

**COUPLED MECHANICAL AND
HYDROLOGIC PROCESSES**