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## The Fracmeter: A New Numerical Method To Evaluate the State of Stress and the Elastic Properties of Rocks

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### ABSTRACT

The fracmeter is a straddle inflatable packer with a spacing fitted with radial displacement transducers. When a hydraulic fracture is created it can be shown that the displacement field around the wellbore becomes anisotropic with a maximum value perpendicular to the fracture plane.

The multiplication of the number of equations relating the radial displacement of the borehole and the pressure makes it possible by an appropriate inverse method to determine with a single test the two horizontal geostatic stresses, the elastic constants of the rock and the fracture azimuth.

### INTRODUCTION

The only method used today to determine certain components of the geostatic state of stress at great depth is the classical hydraulic fracturing!

The usual techniques of geomechanics like flat jack<sup>2</sup>, sectionnal pressimetric test<sup>3</sup> overcoring<sup>4</sup> can be only used for small depths (less than 10 m).

None of these methods allows to determine the elastic properties of rocks and to evaluate the fracture azimuth it's often necessary to perform several tests<sup>3-5</sup>

In a recent paper<sup>6</sup>, we have proposed a new cell so-called "the rockmeter", the purpose of which is to combine two essential elements on the same device: the uniform radial pressurization of the borehole above the rupture point using a rubber inflatable packer and the measurement of the associated deformation of the borehole for different azimuths using displacement transducers fixed on the packer itself.

This technique has a technological drawback at great depth: the bad mechanical behaviour of the rubber sleeve at the transducer locations. So the authors have imagined a comparable technique to use the same method at great depth: "The fracmeter".

### DRAWBACKS OF THE CLASSICAL MINIFRAC METHOD (Figure 1)

We consider an infinite elastic homogeneous and isotropic medium (elastic modulus  $E$  and Poisson's ratio  $\nu$ ) subjected to two principal horizontal stresses  $\sigma_1$  and  $\sigma_2$  ( $\sigma_1 > \sigma_2$ ). The maximum principal component of the stress tensor is assumed to be vertical and parallel to the axis of the well (radius  $R$ ). We will suppose for the present that the rock is impermeable.

Between a straddle inflatable packer, we will apply an increasing pressure  $P$ . For a specific value  $P_R$  of the pressure, a symmetrical fracture will initiate at the edge of the borehole when the minimum tangential component  $\sigma_{\theta\theta}$  (compressions are supposed as positive) reaches a critical value  $R^T$  (the traction resistance of the rock). According to the proposed criterion rupture will occur if:

$$\text{MIN}(\sigma_{\theta\theta}) = -R^T \quad (R^T > 0) \quad (1)$$

It can be easily shown<sup>7</sup> that the relation (1) can be written:

$$P_R = 3\sigma_2 - \sigma_1 + R^T \quad (2)$$

Once initiated, the fracture will propagate in a direction perpendicular to  $\sigma_2$ .

When the pumping is stopped, the fracture becomes to close first quickly and after more and more slowly. After complete closing, the pressure returns to the reservoir pressure.

The instantaneous shut in pressure (ISIP) corresponds to an inflexion point in the shut-in curve (figure 2) theoretically equal to the pressure at which the fracture closes.

Considered as a good estimation of  $\sigma_2$ , the ISIP is often difficult to evaluate. A lot of techniques exist but they generally give very scattered results<sup>8-9</sup>.

If after shut in and bleed off a repressurization occurs, it will reopen the fracture for a specific value  $P_0$  such as :

$$P_0 = 3 \sigma_2 - \sigma_1 \quad (3)$$

This formula is only valid if  $\sigma_1 < 3 \sigma_2$  which is often the case of petroleum depth except in very tectonic regions.

The classical minifrac curve is often very difficult to analyse (specially the determination of ISIP which always causes a lot of doubts). It does not give any information on elastic properties of rock and fracture azimuth.

#### THE IDEA OF THE FRACMETER

Imagine now the same tool with displacement transducers in different azimuths into the pressurized room. Firstly as long as the rupture does not occur, the displacement field at the wellbore is uniform and the radial displacement  $U_R$  is related to the pressure  $P$  and the shear modulus  $G = E/2(1 + 2\nu)$  by the relation

$$G = \frac{PR}{2 U_R} \quad (4)$$

This elastic phase allows to measure  $G$ . Secondly, when the fracture is created the displacement field becomes anisotropic and it will be maximum in a direction perpendicular to the fracture. It's then possible to evaluate fracture azimuth. Thirdly, during shut in, the transition between the opened and the closed fracture could be observed in the displacement field itself.

Finally, when the fracture is reopened, if only a small volume is pumped into the fracture, it will not significantly propagate. The problem can then be considered as linear elastic. Then, the displacement field around the borehole can be expressed as follow :

$$U_R(\theta) = F(\sigma_1, \sigma_2, P, R, L, E, \nu, \theta) \quad (5)$$

Knowledge of relation (5) for a certain number of different azimuths will therefore allow one, by

any inverse method to determine in a single test  $\sigma_1, \sigma_2, E, \nu$  and  $L$ .

However the complexity of the boundaries (singularities at the fracture - borehole contacts), prevents the relation (5) from being solved analytically.

We have therefore decided to use a numerical model, based on the "displacement discontinuity method"<sup>10</sup>. This consists in meshing the limits of the zone within which the stresses distribution is being studied, into a certain number of segments, on which the mechanical boundary conditions are constant.

Thus a system of linear equations is established where the unknowns (the discontinuities within the displacement field between each segment) are calculated in such a way as to satisfy the boundary conditions. The symmetry of the problem posed, allows numerical treatment using a single borehole quarter modelled by 10 segments. The fracture itself is also represented by segments, placed on the axis of the maximum principal stress  $\sigma_1$ .

In the following computations, all of the segments are loaded with the same pressure ( $P$  well =  $P$  fracture).

#### RESULTS OF THE DIRECT PROBLEM

The deformation should not take into account the borehole deformation due to geostatic stresses anterior to the pressurization, not monitored during the test. The displacements recorded during the test are the result of the operation shown in figure 3, where the initial state is that of stressed but not pressurized borehole.<sup>11</sup>

The typical curve of figure 4 shows clearly our assumptions : in comparison with a non-fractured well (dashed lines) the displacement field  $U_R$  at the wellbore, is strongly anisotropic with minimum and maximum values respectively in a direction parallel and perpendicular to the fracture (ie. perpendicular to  $\sigma_2$ ).

A parametric study shows that the anisotropy of the displacement field decreases if  $\sigma_2$  is not far from  $\sigma_1$ . Figures 4a and 4b show that the influence of the length of the fracture on the displacement field at the wellbore is very important and its anisotropy increases a lot with  $L$ . It's supposed for these computations that the fracture length remains constant during reopening (no propagation). In a future development of the program, it will be absolutely necessary to take into account fracture propagation (which will add an other parameter, the surface energy), as well as fluid flow into the porous medium.

#### THE INVERSE PROBLEM

The complete problem has in fact (considering no fracture propagation during reopening) five unknowns :  $\sigma_1, \sigma_2, E, \nu$ , and the fracture azimuth. Without taking into account relation (2) we can reduce considerably the number of unknowns :

- The linear part prior to fracturation gives G (relation 4).
- The instantaneous shut in pressure gives an estimation of  $\sigma_2$ .
- The fracture reopening pressure gives a relation between  $\sigma_1$  and  $\sigma_2$  (relation 3).
- The anisotropy of the displacement field makes it possible to evaluate the azimuth of the fracture.

As classical relations of hydraulic fracturing are often difficult to estimate, we will consider that two unknowns remain for the inverse problem ( $\sigma_1$  or  $\sigma_2$ , E or  $\nu$ ) which will be solved by resolution of equation (5) for different values of  $\theta$ . The inverse problem can be understood as follows :

Given a physical system determined by a vector  $\vec{m}$ , a vector  $\vec{d}$  and a non-linear operator g so that :

$$\vec{d} = g(\vec{m}) \quad (6)$$

Finally we will consider the vector  $\vec{d}_0$  known as the "data vector". In the specific problem of hydraulic fracturing,  $\vec{m}$  is the unknown vector (E or  $\nu$ ,  $\sigma_1$  or  $\sigma_2$ ) and  $\vec{d}_0$  is the displacement vector measured at different points of the borehole. The operator g represents the displacement discontinuities method. The direct problem, thoroughly developed in the preceding paragraph consisted in calculating  $\vec{d}$ , knowing  $\vec{m}$ . The inverse problem has the objective of calculating  $\vec{m}$  so that :

$$\vec{d}_0 = g(\vec{m}) \quad (7)$$

The resolution has been realised by an optimization method<sup>12</sup>. It is based on least square method and consists to find a final estimation of  $\vec{m}$  such as :

$$\|g(\vec{m}) - \vec{d}_0\| \text{ is minimum} \quad (8)$$

The algorithm only converges if the final solution  $\vec{m}^*$  is not too far from the first estimation  $\vec{m}_0$ . So it's necessary for  $\vec{m}^*$  to be such that :

$$\|\vec{m}^* - \vec{m}_0\| \text{ is minimum} \quad (9)$$

The solution to the inverse problem is the model  $\vec{m}^*$  which minimizes the expression :

$$S = 1/2 \|g(\vec{m}) - \vec{d}_0\|^2 + \|\vec{m} - \vec{m}_0\|^2 \quad (10)$$

## RESULTS

The displacements calculated by the "displacement discontinuities" (corresponding to E = 250,000 bar,  $\nu = 0.25$ ,  $\sigma_1 = 500$  bar,  $\sigma_2 = 250$  bar, P = 300 bar and L = 20 cm) were injected into the inverse program like a vector do experimentally determined.

Figure 5 represents the iso-S (see equation 10) in a plane ( $\sigma_2$ , E). If the first estimation  $\vec{m}_0$  is too far from the exact solution, there cannot be any convergence specially if the estimated parameters are both much smaller or much greater than the exact solution (starred zones of figure 5). On the contrary if one of the parameters is much smaller and the other one is much greater, the convergence of the system remains much more possible. So the first estimation must be given in this sense.

In a second step, we have not injected in the program the exact values of displacements of figure 4, but a solution for which a voluntary error is affected to each value of displacement. The data are given in the table 1.

These results show that even if the data are of bad quality (error of 20%) the final solution obtained is not very precise (S = 156,528) but of an order of magnitude not completely erroneous. For 10 microns of error (case T3), the quality of the solution is much closer from the exact one (E = 250,000 bar,  $\sigma_2 = 250$  bar). For errors of 5 microns, the exact solution is approached from 3 per cent.

## CONCLUSIONS

It has been shown in this paper that on a theoretical point view the measurement of the borehole deformation during hydraulic fracturing allows to multiply the number of equations between the different parameters. The direct problem solved numerically by the "displacement discontinuity method" has proved that it's a very simple way to determine fracture azimuth, the displacement being maximum in a direction perpendicular to the fracture's plane. The optimization inverse method used to evaluate the parameters but also the fiability of the solution gives very good results in a lot of cases even if the first estimation injected in the model is not very close to the final solution.

If important experimental errors on displacements exist, the final solution is not completely erroneous and of a correct order of magnitude, the only case of no convergence being when all the parameters firstly estimated are too small or too big in comparison with the exact solution.

In the future, the research will be conducted in two ways :

- It is absolutely necessary to perform experimental tests firstly in laboratory before building an "in situ tool" for great depths.

Secondly, to improve the mathematical model, considering both the propagation of the fracture during reopening and the fluid flow through the porous medium.

Although the method is attractive these two points are needed to prove definitively its validity.

REFERENCES

1. Hawks and Fairhurst (1969) "In situ stress determination by means of hydraulic fracturing". 11th symposium on rock mechanics June 1983 Berkeley pp 559-584.
2. Grossman and Camara (1986) "About the rock stress measurement using the LFJ technique" International symposium on rock stress and rock stress measurements - Stockholm 1-3 september 1986. Centek publishers.
3. Charlez Ph. (1983) "Détermination de l'état de contrainte dans les massifs rocheux élastiques et peu perméables". Thèse de docteur ingénieur IPG Paris.
4. Buyle-Bodin F. (1980) "Mesure des contraintes in situ dans les massifs rocheux". Thèse de docteur ingénieur - Institut de physique du globe Paris. Published by BRGM - Rapport 81SGN 254-GEG.
5. Cornet F.H. and B. Valette (1984) In situ stress determination from hydraulic injection test data ; J of Geoph Res Vol. 89 nb B 13 pp 11527-11537.
6. Charlez Ph., Saleh K., Despax D. and Julien Ph. (1986) "A new way to determine the state of stress and the elastic characteristics of rock massive". Int. Symp. on rock stress and rock stress measurements. Stockholm 1-3 september 1986. Centek publishers.
7. Jaeger J.C. and Cook NWG. (1969) "Fundamentals of rock mechanics" Chapman and Hall - London.
8. Hamoût R.L. and M. Kuriyawa (1982) "Measurement of ISIP in cristalline rocks". Proceed of workshop on hydraulic fracturing stress measurement". VS Geol. Sur. openfile rep. 82-1075 pp 394-403.
9. Roeg ers J.C. and Mo Lennon J.D. (1983) "Do ISIP accurately represent the minimum principal stress ?". Hydraulic fracturing stress measurements pp 79-85. National academy press.
10. Crouch S.L. (1976) "Solutions of plane elasticity problems by the displacement discontinuity method". Int. J. for Numerical methods. Vol. 10, pp. 301-343.
11. Saleh K. (1985) "Détermination de l'état de contraintes et des propriétés élastique d'un massif rocheux par inversion des données récoltées lors d'un essai de fracturation pressiométrique". Thèse de docteur ingénieur ECP.

12. Tarantola A. et Valette B (1982) "Generalized nonlinear inverse problems solved using the least square criterion". Rev. Geogh., Space, physiol. Vol. 20 n° 2, p 219-232.

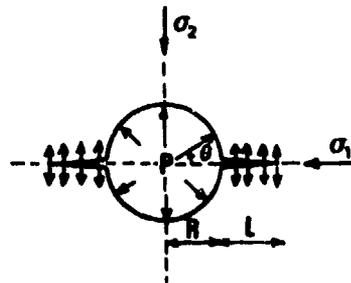


Figure 1 - Geometry of problem

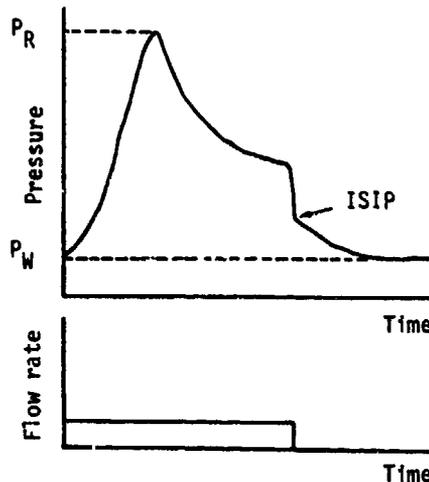


Figure 2 - Classical minifrac curve

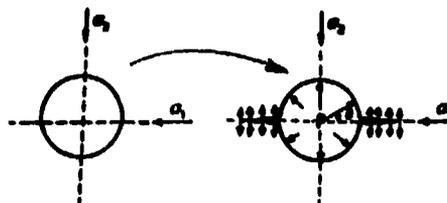


Figure 3 - Initial and final state

$\theta$ (degree)	Exact solution (micron)	Error introduced (micron)			
		$T_1$	$T_2$	$T_3$	$T_4$
4.5	58	0	- 5	- 10	- 20
13.5	81	0	- 5	- 10	- 20
22.5	101.6	0	+ 5	+ 10	+ 20
31.5	118.7	0	+ 5	+ 10	+ 20
40.5	133	0	- 5	- 10	- 20
49.5	144.8	0	+ 5	+ 10	+ 20
58.5	154.1	0	+ 5	+ 10	+ 20
67.5	161.1	0	- 5	- 10	- 20
76.5	165.5	0	+ 5	+ 10	+ 20
85.5	167.8	0	+ 5	+ 10	+ 20
Results	$E$ (bar)	250 000	256 000	266 000	287 000
	$\sigma_2$ (bar)	250	244	236	219

TABLE 1 - Results of inverse problem

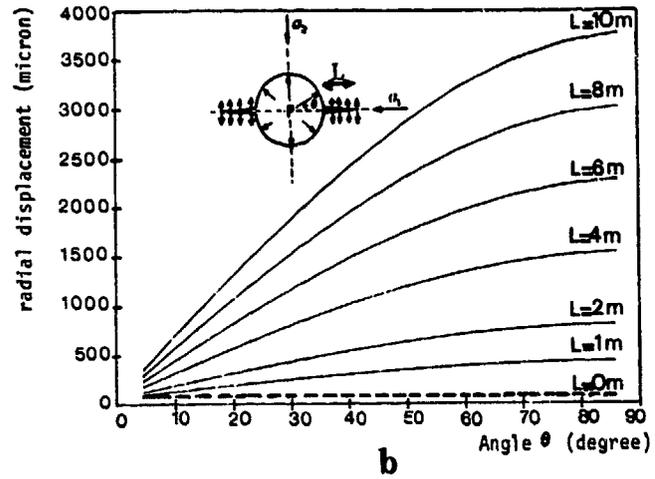
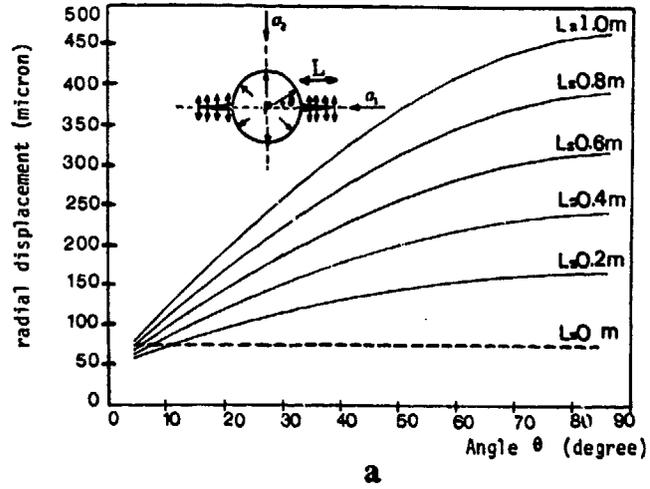


FIGURE 4 - Results of the direct problem.

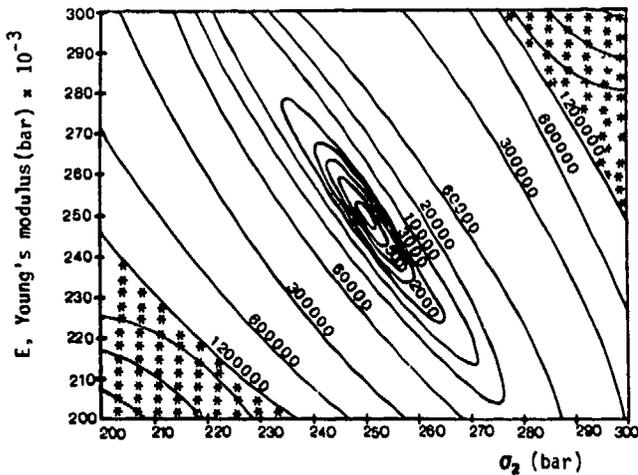


FIGURE 5 - Results of the inverse problem.