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Interpretation of Hydraulic Fracture Parameters by Inversion of Pressure Curves

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In this paper a new method of minifrac interpretation is developed which considers minor principal stress and fluid loss coefficient as unknowns. First, the direct problem is clearly defined by applying the contained model of Nordgren–Nolte (propagation and shut-in phases). Sensitivity of the model is studied, and it is shown that the stress has a great influence on propagation and fluid loss coefficient on shut-in. Second, an inverse method which consists of minimizing a function of the parameter space is proposed. Finally, the inverse method is applied to an actual case and it is shown that a very good approximation of σ_3 and C_w can be obtained.

INTRODUCTION

Over the last few years, the need to develop petroleum reservoirs of increasingly low permeabilities has meant that hydraulic fracturing has gained significantly in importance. In particular, selective fracturing of thin, stratified zones, separated by non-producing interbeds, has made the problem of containment a crucial one. The numerous studies undertaken since the end of the '70s have led to the development of increasingly sophisticated programs [1–3].

These 3-D models have clearly demonstrated that there are three basic parameters which alone govern the extension of the fracture (vertical and horizontal extension): the minor principal component of the stress tensor, the fluid loss coefficient of the various layers, and the viscosity of the fracturation fluid.

For a long time authors have devoted themselves to interpreting the rupture, propagation and closure in terms of stress and the fluid loss coefficient, but the methods which exist today are still very empirical [4–8]. The purpose of this paper is to propose a new method of interpretation of propagation and shut-in pressures, based on an accurate comprehension of the direct problem than on its mathematical inversion.

The contained Nordgren model was selected with the intention of subsequent inversion for the following reasons. First of all, the shape of the contained fracture is the closest to the "petroleum model"; secondly, it is the one (except for 3-D models) for which the pressures are the most correct; finally, the numerical algorithm is sufficiently flexible to hope for a satisfactory convergence throughout the parameter space. It appeared essential to us to review the fundamentals before going to the question of inversion.

DESCRIPTION OF THE NORDGREN MODEL [9]

This model is based on the following hypotheses (Fig. 1):

—The fracture has a constant height h.
—The flow is 1-D. In other words, in all the vertical sections, the pressure $p(x, t)$ is constant.
—Each vertical section is in a state of plane deformations, so that the fracture opening $w(x, z, t)$ depends only on the pressure $p(x, t)$ in this section.

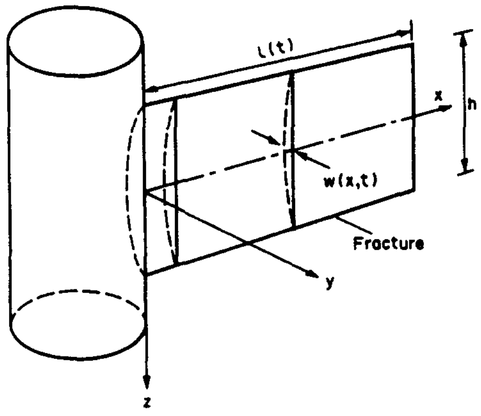

Fig. 1. Nordgren contained model.

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—The fracture is perpendicular to the minor principal component of the stress tensor σ_3. Having accepted these hypotheses, it is possible to demonstrate that any vertical section is an ellipse whose equation is [10]:

$$w(x,z,t)=\frac{1-\nu}{G}(h^2-z^2)^{1/2}\Delta p(x,t)\quad\left[-\frac{h}{2}\leqslant z\leqslant\frac{h}{2}\right],\tag{1}$$

where G and ν are, respectively, the shearing modulus and the Poisson's ratio of the rock. $\Delta p(x,t)$ represents the pressure loss in the fracture:

$$\Delta p(x,t)=p(x,t)-\sigma_3.\tag{2}$$

—Flow is laminar, so that the flow rate $q(x,t)$ in all parts of the fracture is the result of the calculation of the pressure loss through an elliptical tube, height h, and maximum thickness $W(x,t)=w(x,z=0,t)$:

$$q(x,t)=-\frac{\pi h}{64\mu}W^3(x,t)\frac{\partial\Delta p(x,t)}{\partial x},\tag{3}$$

where μ is the dynamic viscosity of the fluid (assumed Newtonian). One part of the volume injected serves to open the fracture whose vertical section $A(x,t)$ is:

$$A(x,t)=\frac{\pi}{4}hW(x,t),\tag{4}$$

whereas another part $ql(x,t)$ leaks into the formation by percolation. The volume balance in the fracture is expressed by the conservative equation:

$$\frac{\partial q(x,t)}{\partial x}+\frac{\partial ql(x,t)}{\partial x}+\frac{\partial A(x,t)}{\partial t}=0,\tag{5}$$

where $ql(x,t)$ is the loss rate through the porous medium, given by the classical law:

$$ql(x,t)=\frac{2hC_w}{\sqrt{t-\tau(x)}}\,\mathrm{d}x.\tag{6}$$

In equation (6), $\tau(x)$ represents the time at which the fracture reaches point x. In other words, the intensity of the loss at a point x decreases with its age [represented by the difference $t-\tau(x)$]. The definition of $\tau(x)$ makes it possible to write that at any moment, the length of the fracture $L(t)$ is such that:

$$\tau[L(t)]=t.\tag{7}$$

By replacing (1) in (3) then (3), (4) and (6) in (5), we get a second-order differential equation in $W^4(x,t)$ which is written:

$$\frac{G}{64(1-\nu)\mu h}\frac{\partial^2W^4(x,t)}{\partial x^2}=\frac{8C_w}{\pi\sqrt{t-\tau(x)}}+\frac{\partial W}{\partial t}.\tag{8}$$

Equation (8) can be solved by finite differences.

INITIAL AND BOUNDARY CONDITIONS

We assume that at the initial moment $t=0$, the fracture does not exist:

$$W(x,t=0)=0\ \forall\ x.\tag{9}$$

At the fracture inlet, the rate is known and equal to $Q(t)$ (it can vary with time). Replacing (1) in (3), we obtain:

$$Q(t)=\frac{-\pi G}{256(1-\nu)\mu}\frac{\partial W^4(x=0,t)}{\partial x}.\tag{10}$$

Finally, it is assumed that the tip of the fracture is completely closed, i.e.

$$W[x=L(t),t]=0.\tag{11}$$

Equation (11) assumes implicitly [according to equation (1)] that the pressure $\Delta p[x=L(t),t]$ at the tip of the fracture is equal to zero. Since we know that the fluid cannot flow as far as the front, the value of the pressure at the fracture tip is below the value of $\sigma_3[\Delta P<0]$, having a magnitude apparently unknown but which can be determined by assuming a type $G=2\gamma$ equilibrium (Griffith criterion). The boundary condition (11) therefore expresses a "pseudopropagation criterion", assuming a $\Delta p[x=L,t]$ always equal to zero. For very short fractures (less then 1 m) the disturbed zone at the tip of the fracture can have considerable impact on the propagation conditions, but as soon as it reaches reasonable dimensions (in practice, almost immediately: 1–5 m) criterion (11) gives an excellent approximation of the problem set.

SHUT-IN PHASE

Shut-in is the specific case for which the injection rate at the well is null. This point was covered by Nolte [4]; below we give a summary of his development. If we take the conservation equation (5) from which we eliminate W, using equation (1), we get:

$$-\frac{\partial q}{\partial x}=\frac{2C_wh}{\sqrt{t-\tau(x)}}+\frac{\pi}{2}\frac{h^2}{E'}\frac{\partial\Delta p}{\partial r}\quad\text{where}\quad E'=\frac{E}{1-\nu^2}.\tag{12}$$

By integrating this equation between 0 and L, which is assumed constant after shut-in (no propagation after shut-in) we get:

$$-q(L)+q(0)=2C_wh\times\int_0^L\frac{\mathrm{d}x}{\sqrt{t-\tau(x)}}+\frac{\pi}{2}\frac{h^2}{E'}\int_0^L\frac{\partial\Delta p}{\partial t}\mathrm{d}x.\tag{13}$$

Because after shut-in the flowrate is null both at the fracture inlet ($x=0$) and tip ($x=L$), the evolution of the pressure at the well can be expressed as:

$$\frac{\mathrm{d}\Delta P}{\mathrm{d}t}=-C_1f(t),\tag{14}$$

where ΔP is the well pressure loss.

$$f(t)=\frac{\sqrt{t_F}}{L}\int_0^L\frac{\mathrm{d}x}{\sqrt{t-\tau(x)}},$$

$$C_1=-\frac{4}{\pi}\frac{C_wE'}{h\beta_s\sqrt{t_F}},\tag{15}$$

with t_F = shut-in time.

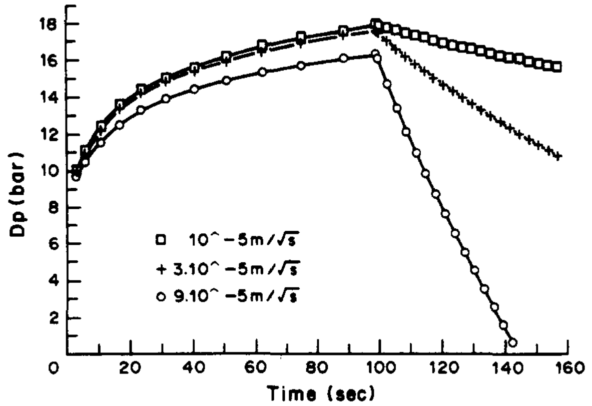

Fig. 2. Influence of fluid loss coefficient on propagation and shut-in pressure.

In expression (15), β_s (called "shut-in coefficient") is the ratio of the average pressure in the fracture and the pressure in the well during fracture closure.

This can only be calculated analytically if the expression of $q(x)$ is taken during closure. In his original paper Nolte assumes this value to be constant, which clearly it is not, as $q(0) = q(L) = 0$ during closure. We will assume, within the scope of this model, that $q(x)$ is parabolic:

$$q(x) = \frac{4Q_0}{L}\left[x - \frac{x^2}{L}\right].$$

This approximation leads to an expression of b_s such that:

$$\Rightarrow \beta_s = \frac{\overline{\Delta p}}{\Delta p(x=0)} = \frac{1}{L^{3/2}} \int_0^L \left(L^2 - 3x^2 + \frac{2x^3}{L}\right)^{1/4} \mathrm{d}x. \tag{16}$$

The numerical integration of the latter equation shows that β_s varies only slightly with L and can be considered as a constant, equal to approximately 0.76, a value which is actually very close to $\beta_s = 0.8$ put forward by Nolte for the case of a constant flowrate.

The integration of equation (14) makes it possible to calculate the evolution of the pressure at the well during closure:

$$\Delta P(t) = P_F - C_1 \int_{t_F^*}^{t} f(t)\,\mathrm{d}t, \tag{17}$$

P_F being the pressure at closure. It should be noted that since $f(t_F)$ is infinite [as $\tau(L) = t_F$], the integration of formula (24) can, in fact, only be applied between $t_F + \Delta t_F = t_F^*$ and t (with ΔT_F small).

INFLUENCE OF THE DIFFERENT PARAMETERS ON THE PHENOMENON

In fact, the point of interest is really the influence of σ_3 and C_w on the pressure–time curves at the well, which forms the basic experimental data. By applying equation (2) the problem can be completely uncoupled.

For C_w constant, and σ_3 variable, the curve moves parallel to itself, as σ_3 is independent of time. The influence of σ_3 is therefore very important. However, for σ_3 constant and C_w variable, sensitivity is not so good, due mainly to the fact that $\Delta p(x = 0, t)$ is generally an order of magnitude below that of σ_3.

Figure 2 shows that $\Delta p(x = 0, t)$ depends only slightly on C_w during propagation, but is much more sensitive to this parameter during closure. In fact, C_w would really have the strongest effect on the length of the fracture during propagation, but as this value cannot be measured, this sensitivity would be of no interest for the inverse model.

This fundamental point shows, therefore, that it is of great importance to model both the propagation and the closure, as the former is sensitive almost exclusively to stress, and the latter to C_w.

THE INVERSE PROBLEM

The inverse problem consists of determining certain parameters which govern the phenomenon, knowing the pressure evolution at the well as a function of time.

If **P** represents this evolution, **p** the parameters to be found (σ_3 and C_w in the present case) and **c** are constants assumed known (E, ν, μ and h), the direct problem can be put in vectorial form:

$$\mathbf{P} = \mathbf{g}(\mathbf{p}, \mathbf{c}) \tag{18}$$

or, when uncoupled σ_3, Δp:

$$P(t) = \sigma_3 + \Delta P(C_w, E, \nu, \mu, h, t), \tag{19}$$

where ΔP is calculated using the Nordgren model.

The inverse problem may be solved by applying one of several methods, but whichever is used, it is a question of minimizing a function S characterizing the difference between the experimental data and the model. In a first phase, it is advisable to investigate the entire parameter space, especially if the problem is strongly non-linear, because several solutions can exist together [11].

Thus, if P_k^m represents the measured value of the pressure at the moment k, and P_{ijk}^c represents the same pressure calculated at the same moment for arbitrary values C_{wi}, and σ_{3j} of the parameters, the solution of the problem consists in determining C_{wi}^0 and σ_{3j}^0 so that:

$$S = \sum_{k=1}^{N} [P_k^m - P_{ijk}^c] \text{ is a minimum,} \tag{20}$$

N being the number of experimental observations.

Therefore, the algorithm consists in meshing the plane C_w, σ_3 and calculating at each node i, j the function $S(C_{wi}, \sigma_{3j})$. The minimum value obtained for S will correspond to the solution C_{wi}^0 and σ_{3j}^0.

SELECTING THE LIMITS OF THE GRID

An *a priori* knowledge of the parameters makes it possible to select the appropriate mesh limits.

First, the upper limit of σ_3 is known accurately, and it corresponds to the minimum P_0 of the experimental

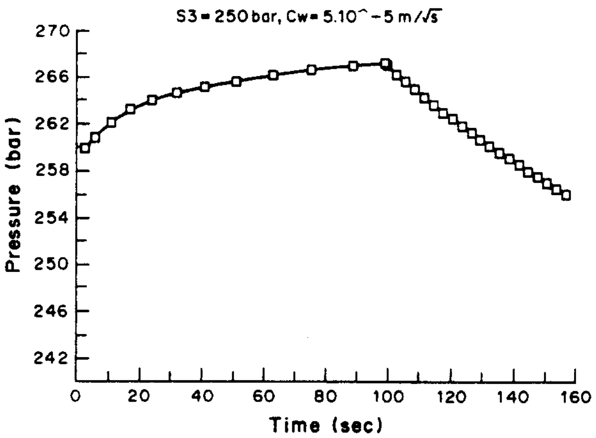

Fig. 3. Theoretical case—direct problem.

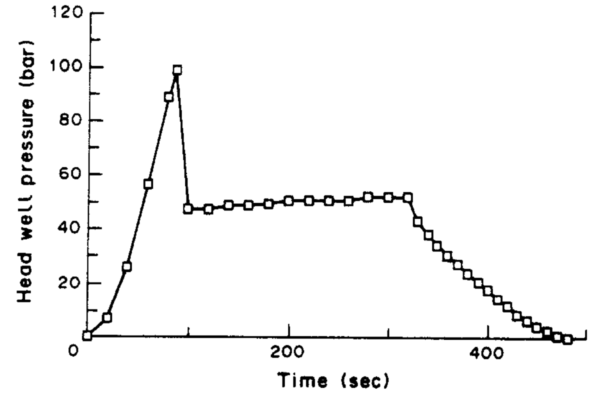

Fig. 5. Minifrac in Persian Gulf.

values (the stress σ_3 cannot be greater than the propagation pressure at the well). Secondly, it is known by having studied the direct problem that the ΔP is rarely greater than 5 MPa. The parameter σ_3 is therefore limited between $[P_0, P_0 + 5\,\text{MPa}]$ where P_0 is the first value of propagation (it should be remembered that in the Nordgren model the well pressure increases with time).

Knowledge of C_w is much poorer as no information is *a priori* available.

However, we know by experience that below a value of $10^{-6}\,\text{m}/\sqrt{\text{sec}}$ the C_w has no more influence on the curve (in other words, it can no longer be measured) and that over a certain value (provided by the model) propagation over a reasonable distance is no longer possible (superior limit of C_w).

RESULTS

The algorithm of inversion was first tested on a theoretical solution corresponding to a minor principal stress of 25 MPa and a $C_w = 510^{-5}\,\text{m}/\sqrt{\text{sec}}$ (Fig. 3). The constants of the model are, respectively, $H = 5$ m, $E = 10^4$ MPa, $\nu = 0.2$, $\mu = 10^{-3}$ Pa.sec.

The shape of the response $S(\sigma_3, C_w)$ is shown in Fig. 4. These curves show that the value of S is indeed minimum for $C_w = 510^{-5}\,\text{m}/\sqrt{\text{sec}}$ and $\sigma_3 = 25$ MPa.

The algorithm thus provides the exact solution in the case of an exact direct problem.

The algorithm was then used to inverse an actual fracturation curve obtained during a minifrac in the Arabian Gulf.

Figure 5 presents the head well pressure as a function of time. To carry out the inversion, only the part located after the fracturating peak is taken into consideration. The constants of the problem are:

$$h = 6.6\ \text{m},$$

$$\mu = 10^{-3}\ \text{Pa.sec},$$

$$Q = 0.002\ \text{m}^3/\text{sec (for the two wings)},$$

$$E = 25{,}000\ \text{MPa},$$

$$\nu = 0.2.$$

The response is represented in Fig. 6. The calculation shows that the closest approximation for the problem is obtained by the two parameters $\sigma_3 = 4.25$ MPa, and $C_w = 0.000073\,\text{m}/\sqrt{\text{sec}}$.

This solution is then used to compare the experimental curve of Fig. 5 with its approximation (Fig. 7). The fitting is most satisfactory. It shows, in particular, that at the end of fall off, when the fracture is completely closed, the slope of the experimental curve drops below the one plotted by the model, which proves that there is a gradual change from a linear to a pseudo-radial flow pattern.

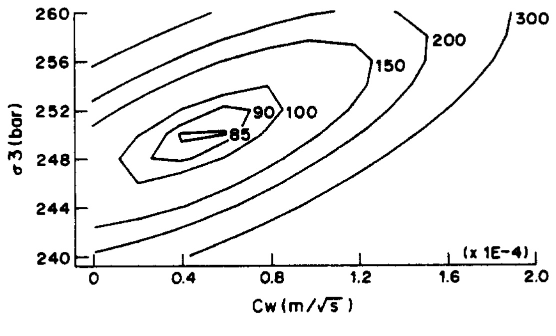

Fig. 4. Response in parameter space (theoretical case).

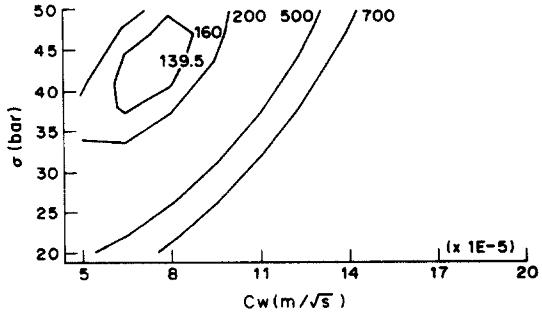

Fig. 6. Response in parameter space (minifrac in Persian Gulf).

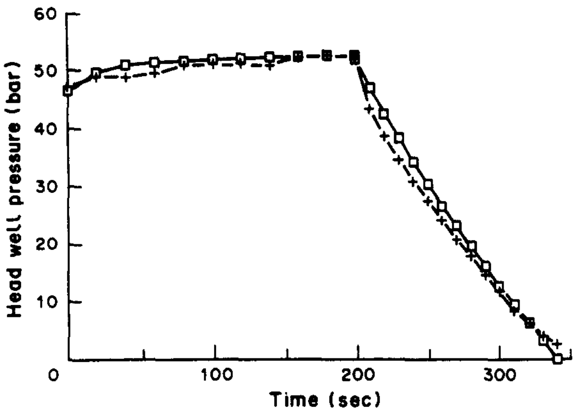

Fig. 7. Experimental and fitted curves (μ = 1 cP).

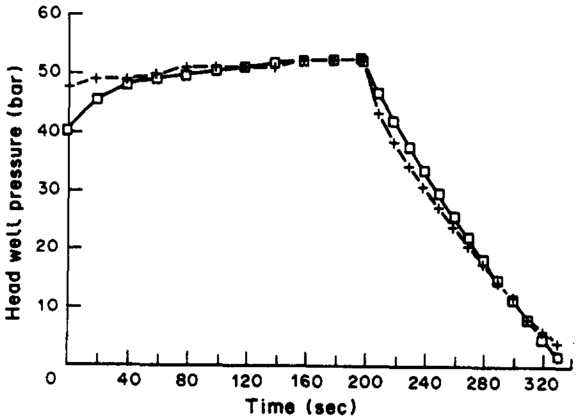

Fig. 8. Experimental and fitted curves (μ = 80 cP).

INFLUENCE OF THE VISCOSITY ON THE INVERSE PROBLEM

So far, the viscosity of the fluid has been considered as one of the constants of the problem. However, in certain cases, it is not well-known, in particular when, for operational reasons, the test has to be run with drilling mud.

Thus we have redone the inversion of the problem with a 810^{-2} Pa.sec viscosity fluid. In this case the inversion gives the following results:

$$\sigma_3 = 2.75\ \text{MPa},$$

$$C_w = 0.000076\ \text{m}/\sqrt{\text{sec}}.$$

The viscosity therefore has a considerable effect on the stress, the ΔP being much higher when the viscosity is high. The comparison between the experimental curve and the recalculated curve (Fig. 8) remains acceptable, except at the beginning of injection (fracture too short compared with the height). Therefore, if more than two parameters are taken as unknowns (e.g. σ_3, C_w, E and μ) the inverse problem offers several solutions and thus becomes insoluble.

CONCLUSIONS

It has been demonstrated in this paper that the interpretation of a fracturing curve can be considered as an inverse problem, and that by the correct choice of a direct model, it is possible to determine two parameters reasonably accurately, whereby the inverse problem contains only one minimum. The solution could, in fact, be refined by applying a method of optimizing [12]. The studied case shows that the stress is very sensitive to the parameters involved during the propagation (especially viscosity), whereas the closure largely depends on the fluid loss coefficient C_w. However, it appears pointless to use such a method to determine more than two parameters if there is no additional information.

The use of flowrate variations during the injection phase could perhaps provide a partial solution to this problem.

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